MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(All sections / Groups)

10 OCTOBER 2016
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 6 printed pages (inclusive of the front page) with 6 questions only. Page 6 is an appendix for the logical equivalence laws and rules of inference. Last two pages are the Standard Normal distribution table.
2. Attempt **ALL SIX** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.
4. Show **ALL** of your working steps clearly.
5. **NO** calculator is allowed in this examination.

QUESTION 1

a) Let $A = \{1,2,3,4,5,6\}$

$B = \{x \in \mathrm{N} \mid x \text{ is divisible by } 6 \wedge x \leq 30\}$

Find

i) $A \Delta B$ [2 marks]

ii) $|\mathrm{P}(A \cap B)|$ [1 mark]

iii) $\mathrm{P}(A \cap B)$ [1 mark]

b) Let $C = \{0,1,2\}$. A binary relation R on set C is defined as

$$R = \{(a,b) \mid a \leq b\} \text{ for all } a,b \in C$$

i) List the elements of R. [1 mark]

ii) Is R reflexive, symmetric and/or transitive? Justify each of your answer. [3 marks]

c) Let f be the following function:

$$f : \{x \mid x \in \mathrm{N} \wedge x \leq 5\} \to \Re, \; f(x) = x + 1$$

i) Determine the range of f. [1 mark]

ii) Is f onto? Justify your answer. [1 mark]

QUESTION 2

a) Show that $\neg[p \vee \neg(p \wedge q)]$ is a contradiction by using a truth table. [2.5 marks]

b) Consider the following predicates with universe of discourse (UOD) = all people:

$A(x)$: x is a junior.

$B(x)$: x is a senior.

$C(x)$: x is enrolled in a music class.

Given the following argument:

No junior or senior is enrolled in a music class. Alice is enrolled in a music class. Therefore, Alice is not a senior.

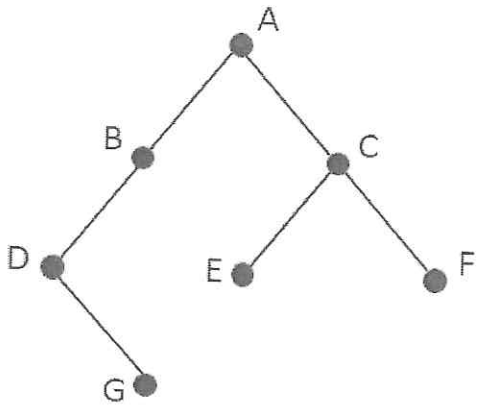
i) Translate the argument into TWO premises and ONE conclusion. [2 marks]

ii) Show that the translated argument in (i) is valid with formal reasoning. Please present all your steps and state clearly each of the laws that you have used in your proof. [5.5 marks]

Continued ...

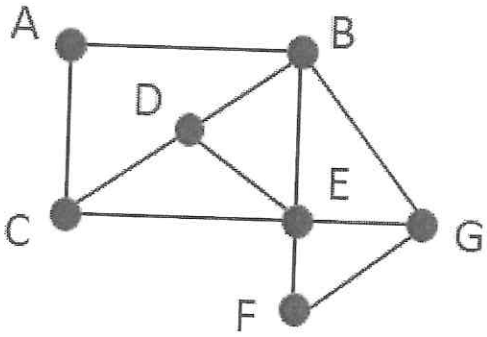
QUESTION 3

a) Write the visitation order for the Pre-order traversal and Post-order traversal on this tree.



[2 marks]

b) Given the following graph:



i) Determine if this graph has an Euler circuit. If yes, exhibit one. If not, explain why not. [2 marks]

ii) Use the breadth first search algorithm to find a spanning tree in this graph. Take vertex **A** as the root and apply alphabetical order if there is more than one choice in the path. Show clearly each step of how the algorithm is performed and present your answer in a table with two columns, the first column is the resulting queue of the search algorithm and the second column is the edges of the resulting spanning tree. [6 marks]

Continued ...

QUESTION 4

a) Use mathematical induction to prove that for all integer $n \geq 1$,

$$2+4+6+\ldots+2n = n^2+n.$$

[7 marks]

b) Show that the average case complexity for $f(n) = 4n^2 + 6$ is $\Theta(n^2)$.

[3 marks]

QUESTION 5

a) Table below denotes the next-state function of a finite-state automaton A having S_0 as initial state and S_2 as the accepting state:

	Next State	
	a	b
S_0	S_1	S_0
S_1	S_1	S_2
S_2	S_1	S_2

i) Construct the state-transition diagram for the A. [4 marks]

ii) To what state does the automaton go when the symbols of the following strings are input to it in sequence, starting from the initial state?

A) *bbaaaba*

B) *ababaaabb*

[2 marks]

b) Suppose P is a set with 3 elements and Q is a set with 4 elements.

i) How many functions can be formed from P to Q? [2 marks]

ii) How many one-to-one functions can be formed form P to Q? [2 marks]

For each of the above, explain your answer.

Continued ...

QUESTION 6

a) Use pigeonhole principle to determine the minimum number of people required to be sure that at least five were born in the same month. Show proper steps. [2 marks]

b) Two factories, A and B are producing a certain electronic component. The probability that a component from the factory A is defective is 3%, and the probability that a component from the factory B is defective is 5%. In a supply of 250 of the parts, 150 were obtained from factory A and 100 from factory B. [**Leave your answer in the simplest fraction form.**]

i) What is the probability that a part chosen at random from the 250 is from the factory A? [1 mark]

ii) What is the probability that a part chosen at random from the 250 is from the factory B? [1 mark]

iii) What is the probability that a part chosen at random from the 250 is defective? [1.5 marks]

iv) If the chosen part is defective, what is the probability that it came from the factory A? [1.5 marks]

c) Suppose the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 $(\text{milliamperes})^2$. What is the probability that a measurement will not exceed 8 milliamperes. [3 marks]

Continued ...

Appendix

List of Rules of Inference

ADD : $p \Rightarrow (p \vee q)$

SIMP : $[p \wedge q] \Rightarrow p$

CONJ : $p \wedge q \Rightarrow (p \wedge q)$

MP : $[(p \rightarrow q) \wedge p] \Rightarrow q$

MT : $[(p \rightarrow q) \wedge (\neg q)] \Rightarrow \neg p$

HS : $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$

DS : $[(p \vee q) \wedge (\neg p)] \Rightarrow q$

RES : $[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$

List of Logical Equivalence Laws

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Double Negation: $\neg\neg p \Leftrightarrow p$

DeMorgan :
(i) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$
(ii) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Domination :
(i) $p \wedge F \Leftrightarrow F$
(ii) $p \vee T \Leftrightarrow T$

Negation :
(i) $p \wedge \neg p \Leftrightarrow F$
(ii) $p \vee \neg p \Leftrightarrow T$

Identity :
(i) $p \wedge T \Leftrightarrow p$
(ii) $p \vee F \Leftrightarrow p$

Commutative :
(i) $p \wedge q \Leftrightarrow q \wedge p$
(ii) $p \vee q \Leftrightarrow q \vee p$

Idempotent :
(i) $p \vee p \Leftrightarrow p$
(ii) $p \wedge p \Leftrightarrow p$

Distributive :
(i) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
(ii) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Associative :
(i) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$
(ii) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$

End of Page.

Table A.3 Standard Normal Curve Areas *(cont.)*

$\Phi(z) = P(Z \le z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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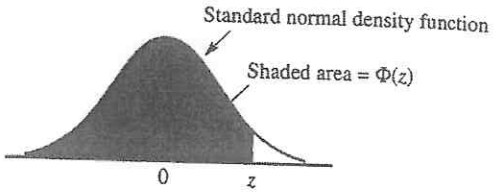
$\Phi(z) = P(Z \le z)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(*continued*)